

2005 Marywood Mathematics Contest

Level II

Sponsored by

SEMI-GROUP

The Student Mathematics Club of

Marywood University

April 2, 2005

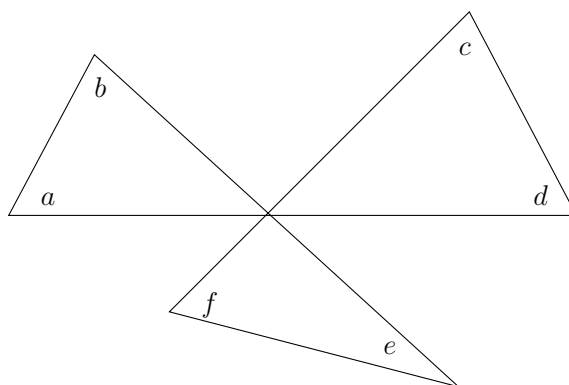
Directions:

1. This exam consists of 40 questions in order of approximately increasing difficulty.
2. Allot your time accordingly. This is a 60-minute test. Do not spend too much time on any one problem. If a question seems to be too difficult, make your best possible guess.
3. Do not worry if you do not finish the test. Your score will be the number of correct responses.
4. On the scantron form provided for you, darken in the space corresponding to the correct answer. Please mark all answers carefully and erase completely when changing an answer. Mark only one answer for each question. Only those answers on the answer sheet will be counted.
5. There is a sheet of blank paper on the last page which you can (carefully) tear off and use as scratch paper.
6. NOTE: In order to ensure uniformity, proctors are NOT allowed to answer any questions pertaining to specific problem content.

Please do NOT open the test until you are told to do so.

- Find the total surface area in square meters of a rectangular solid whose length is 7 meters, width is 6 meters, and depth is 3 meters.
A. 32 B. 162 C. 81 D. 126 E. None of these.
- A fair coin is tossed 3 times. What is the probability of obtaining three heads?
A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. $\frac{1}{6}$ E. None of these.
- The average of two numbers is xy . If one number is equal to x , the other number equals
A. y B. $2y$ C. $xy - x$ D. $2xy - x$ E. None of these.
- A room has six doorways. In how many ways can a person enter and leave the room without using the same door?
A. 5 B. 6 C. 11 D. 25 E. None of these.
- If the operation \oplus is defined by $x \oplus y = 3y + y^x$, then $2 \oplus 5 =$
A. 10 B. -10 C. 40 D. -26 E. None of these.
- Starting with the point $(5, 1)$ in the Cartesian plane, reflect it across the x -axis, then rotate it 180° around the origin, and then finally translate it vertically up by 2 units. The final point is:
A. $(7, -1)$ B. $(5, -3)$ C. $(-5, 3)$ D. $(-5, -1)$ E. None of these.
- What is the area of the region enclosed by $|x| + |y| = 4$ for $-4 \leq x \leq 4$.
A. $16\sqrt{2}$ B. 32 C. $40\sqrt{2}$ D. 48 E. None of these.
- The graphs of $y = x^2 - 1$ and $y = 2x - 2.1$ intersect at how many points?
A. 0 B. 1 C. 2 D. 3 E. None of these.
- A car traveled 3 miles south, 9 miles east, and then another 9 miles south. What is the straight-line distance (in miles) between the point where the car started and where it stopped?
A. 21 B. $6\sqrt{3}$ C. $7\sqrt{3}$ D. 15 E. None of these.

10. What is the coefficient of x^8 in the polynomial $(x + 3)^{10}$?
- A. 405 B. 45 C. 90 D. 810 E. None of these.
11. Which of the following statements is true?
- A. Exactly 1 of these statements is false.
 B. Exactly 2 of these statements are false.
 C. Exactly 3 of these statements are false.
 D. Exactly 4 of these statements are false.
 E. Exactly 5 of these statements are false.
12. What is the sum of the angles a, b, c, d, e and f ?



- A. 90° B. 180° C. 270° D. 315° E. 360°
13. Simplify
- $$\frac{1 + \cot^2 x}{\csc^2 x + \sec^2 x}$$
- A. $\sin^2 x$ B. $\tan^2 x$ C. $\cot^2 x$ D. $\cos^2 x$ E. None of these.
14. If $\sin \theta = 1/3$ and $\pi/2 < \theta < \pi$, what is $\sin 2\theta$?
- A. $2/3$ B. $2\sqrt{2}/9$ C. $4\sqrt{2}/9$ D. $-2\sqrt{2}/9$ E. $-4\sqrt{2}/9$
15. A two-inch cube ($2 \times 2 \times 2$) of silver weighs 3 pounds and is worth \$320. How much is a three-inch cube ($3 \times 3 \times 3$) of silver worth?
- A. \$480 B. \$600 C. \$900 D. \$1080 E. None of these.

16. What is the radius of the circle with equation $x^2 + y^2 - 8x + 2y + 8 = 0$?

- A. 3 B. 4 C. 8 D. 9 E. None of these.

17. What is the value of

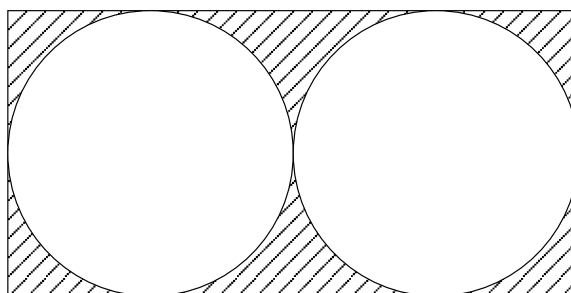
$$\sum_{k=1}^{99} \log_{10} \frac{k}{k+1}$$

- A. $\log_{10} 2$ B. 2 C. $\log_{10} \frac{1}{2}$ D. -2 E. None of these.

18. The non-zero real numbers a, b, c, d have the properties that $\frac{ax+b}{cx+d} = 1$ has no solution in x and that $ad - bc \neq 0$. What is the value of $\frac{a^2}{a^2+c^2}$?

- A. 0 B. $1/2$ C. 1 D. 2 E. a negative number

19. The congruent circles below are tangent to the sides of the rectangle and tangent to each other. If the radius of each is equal to 2, what is the area of the shaded region?



- A. $32 - 8\pi$ B. $8 - 4\pi$ C. $16\pi - 8$ D. $16(\pi - 1)$ E. None of these.

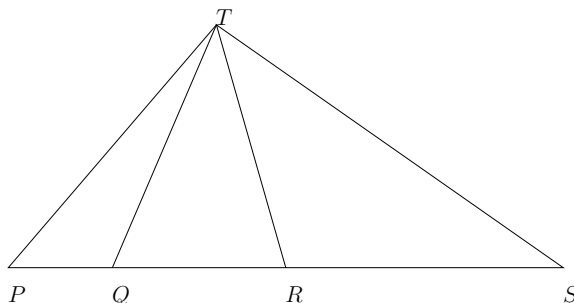
20. The product of three consecutive integers is 33 times the sum of the three integers. What is the product?

- A. 330 B. 660 C. 990 D. 1120 E. None of these.

21. Maggie has 2 quarters, 3 nickels, and 3 pennies. If she selects 3 coins at random, what is the probability the total value is exactly 35 cents?

- A. $3/56$ B. $2/28$ C. $5/56$ D. $3/28$ E. $7/56$

22. If the area of $\triangle PRT$ is 10 cm^2 , and the area of $\triangle QST$ is 16 cm^2 , and the area of $\triangle PST$ is 20 cm^2 , determine the area of $\triangle QRT$.



- A. 5 cm^2 B. 6 cm^2 C. 8 cm^2 D. 16 cm^2 E. None of these.
23. If in $\triangle ABC$, the three sides AB , BC and CA are of lengths 7, 8, 9 respectively, what is $\cos \angle ABC$?
- A. $2/7$ B. $11/21$ C. $2/3$ D. $1/2$ E. None of these
24. The line through the points $(m, -9)$ and $(7, m)$ has slope m . What is the value of m ?
- A. 3 B. $-7/9$ C. 16 D. 5 E. None of these.
25. Find
- $$\sum_{n=1}^{10} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{1024}.$$
- A. $\frac{1023}{1024}$ B. 0.9 C. $\frac{2047}{2048}$ D. $\frac{511}{512}$ E. 1
26. Which of the following statement is FALSE?
- A. The sum of 3 consecutive integers is always divisible by 3.
 B. The sum of 4 consecutive integers is always divisible by 4.
 C. The sum of 5 consecutive integers is always divisible by 5.
 D. The sum of 2005 consecutive integers is always divisible by 2005.
 E. None of the above.
27. How many even numbers can be formed from the digits 1, 2, 3, 4 and 5 using each of the digits exactly once?
- A. 24 B. 36 C. 48 D. 120 E. None of these.

28. There exist positive integers x , y , and z satisfying

$$29x + 30y + 31z = 366.$$

What is the value of $z - x$?

- A. 6 B. 3 C. 7 D. -3 E. -6
29. What is the last digit in the number 13^{2005} ?
- A. 1 B. 3 C. 5 D. 7 E. 9
30. Find the area of the circle inscribed inside the triangle with all three sides of length 12.
- A. 12 B. $4\sqrt{3}\pi$ C. $36\sqrt{3}$ D. 12π E. $4\sqrt{3}$
31. If two real numbers x and y are such that xy , $\frac{x}{y}$ and $x + y$ are all equal, then $x - y =$
- A. $-3/2$ B. $3/2$ C. $1/2$ D. $-1/2$ E. None of these.
32. How many different real solutions does the following equation have?

$$x^3 - 3x^2 + 7x - 5 = 0$$

- A. 0 B. 1 C. 2 D. 3 E. 4
33. Let x_1 and x_2 be the real solutions of the quadratic equation $x^2 + bx + c = 0$. If $x_1 - x_2 = 4$ and $x_1^2 + x_2^2 = 40$, then b must be equal to
- A. 8 B. 4 C. 12 D. 8 or 4 E. 8 or -8
34. A sequence $\{a_n\}$ is defined by $a_1 = 6$ and $a_n = 6 - a_{n-1}$. Find a_{2005} .
- A. 2005 B. 2008 C. -2005 D. -2002 E. None of these.
35. How many decimal digits are there in the number 2005^{2005} ?
- A. 2005 B. 6621 C. 6620 D. 2004 E. None of these.
36. How many different prime numbers are factors of N if

$$\log_2(\log_3(\log_5 N)) = 7$$

- A. 3 B. 4 C. 7 D. 128 E. None of these

37. If a ball and a cube have the same volume, what is the ratio between the surface areas of the ball and the cube?
- A. 1 B. $\sqrt[3]{\frac{3}{4\pi}}$ C. $\sqrt[3]{\frac{9}{16\pi^2}}$ D. $\sqrt[3]{\frac{9\pi}{16}}$ E. $\sqrt[3]{\frac{\pi}{6}}$
38. What is the minimum number of real root(s) a polynomial of degree 3 can have?
- A. 0 B. 1 C. 2 D. 3 E. 4
39. You are given 5 cubes that look and feel exactly the same, with one of them having a slightly different weight from the other four. Using only a two-pan balance, what is the minimum number of weighings needed to find the cube that's different from the others in weight in the worst case?
- A. 1 B. 2 C. 3 D. 4 E. 5
40. A line passing through the point $(3, 2)$ is tangent to the circle $x^2 + (y - 1)^2 = 5$, what is the slope of the line?
- A. The slope must be 2.
B. The slope must be $-\frac{1}{2}$.
C. The slope must be $\frac{1}{3}$.
D. The slope must be 2 or $-\frac{1}{2}$.
E. The slope must be 2 or $\frac{1}{3}$.